The Criswell System of Sunrise/set Prediction

June 1987

A Cautionary Word:
Please read one page at a time.
Do Not skip ahead.

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Chapter I. Introduction

In Which the Problem is set before the Reader in Clearly Defined Terms with Visual Aids

To predict ~~the~~ time of sunrise and sunset it is necessary to compute the "semiduration of sunlight" from one's position of observation. This is the time it takes the sun to travel from the eastern visible horizon to the observer's local meridian or from the local meridian to the western hor. This meridian is a great circle of the Celestial Sphere passing through the north and south celestial poles and the observer's zenith and is called the Celestial Meridian (CM). The sun is on the CM at noon Local Apparent Time (LAT).

Since semiduration will vary with the observer's latitude and the sun's declination, these two quantities must be variables in any system of equation for ~~predicting~~ computing it. In these notes d = declination and L = latitude.

If we ignore for the moment dip, refraction, parallax, irradiation and semidiameter (Chapter IV and appendix A), and define semiduration as the sun's passage from the Celestial Horizon (CH) to the Celestial Meridian (CM), then semiduration can be converted to an angular measurement called the meridian angle (t).

The CH is a plane passing through the center of the earth and parallel to a plane tangent to earth's curvature at the point of observation.

The best way to show t is to draw a diagram on the CM (a cross section of the celestial sphere cut along the CM and seen from outside the celest. sphere — the God's-Eye View (GEV)), with the CH dividing it in half horizontally. t is the angle at the elevated pole between the sun's meridian and the CM.

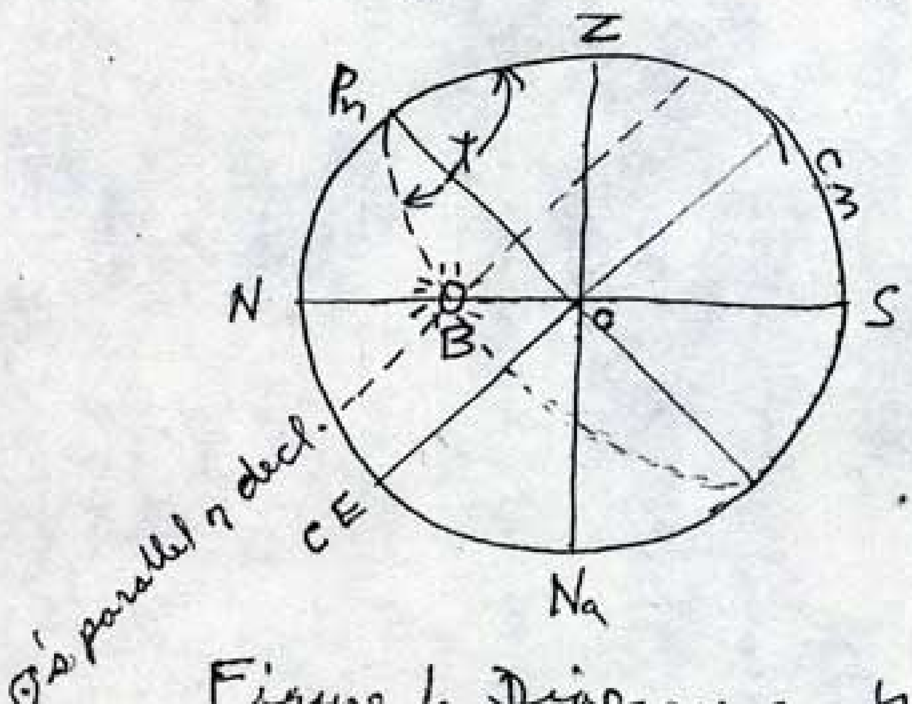


Figure 1. Diagram on the celestial meridian

P_n = North celest. pole (our "elevated" pole)
P_s = South " "
Z = Zenith
Na = Nadir
B = the sun
N = the north point of the horizon
S = " south " " " "
CE = celest. equator
O = the observer, and E-W line seen end-on.

Note that this is only an enlarged view of our original diagrams for amplitude computation and nothing to get alarmed about. Here is the meridian angle t seen from above the celestial pole:

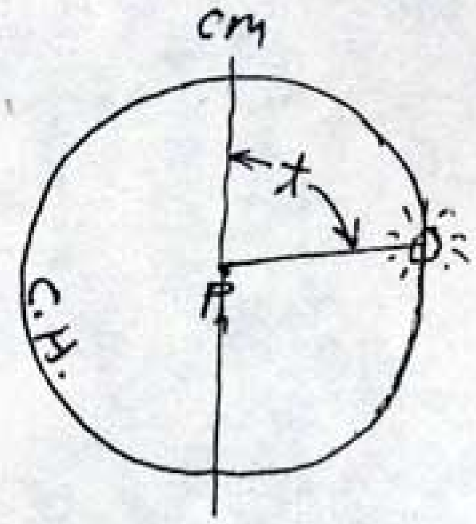


Figure 2.

ZP_nB forms 2 sides of the Navigational Triangle (Bowditch) or Astronomical Triangle (your trig book). The other side is ZB and the angle P_nZB is the sun's azimuth (Z).

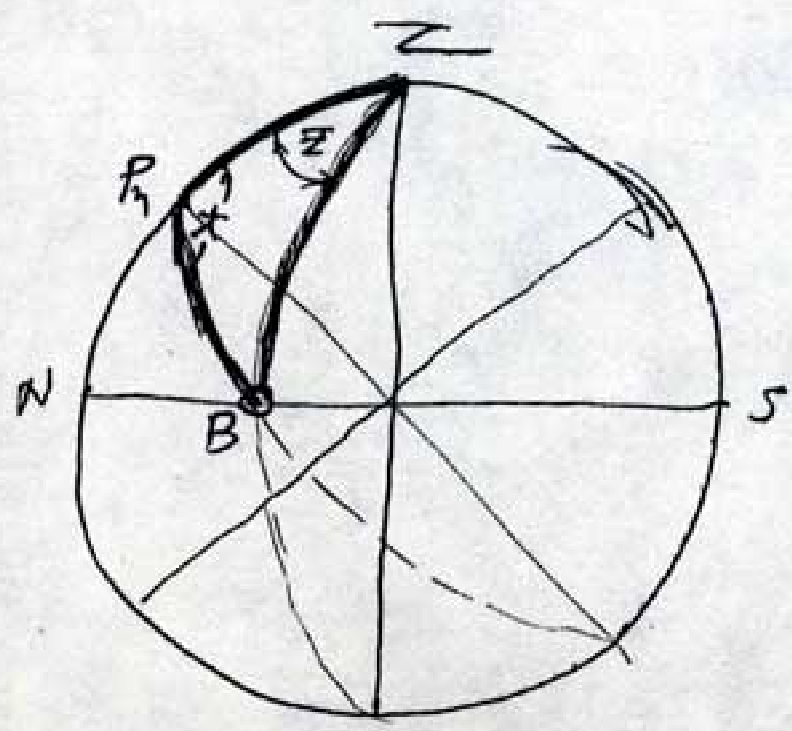


Fig. 3 The Navigational Triangle

The amplitude, $\bar{A}$, is $180° - Z$ or angle BZO (Fig 3). The observer's latitude is equal to the altitude of the elevated pole, NOP_n, or the arc NP_n, making side P_nZ of the ~~NT~~ Navig. Triangle (NT) the Co-latitude. ("Co-" always refers to the "complement," or 90° minus the angle or side.)

The other known quantity is the sun's declination, d. Fig. 1 shows that side P_nB is the co-declination.

So we know L and d and have to solve for t and Z.

Chapter II.

In Which a Formula for t and Z is Derived

The problem is to divide the oblique spherical triangle $P_n BZ$ (the NT/AT) into two right spherical triangles that can be solved by Napier's Rule. At least that's been the approach of the writer cited by Bowditch and apparently of the writers of your Trig book as well. I tried a few of those myself (see earlier letters) but without much success. ~~The idea is to~~ In the end I used a different approach. It's original and therefore suspect, but it seems to work. Understanding the next paragraph.. Wait, let me emphasize that.

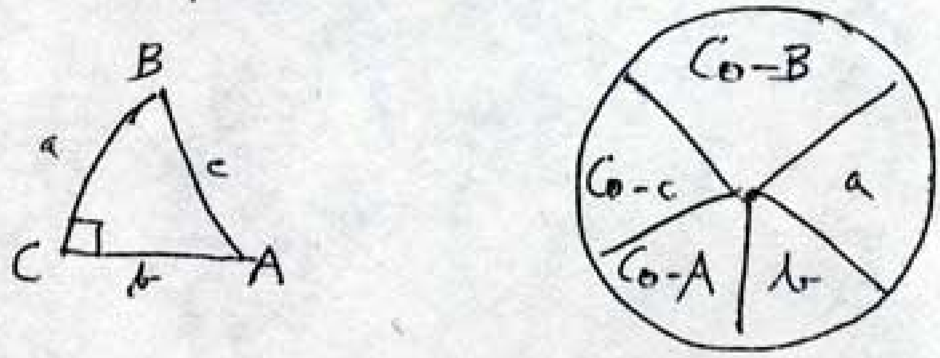
> WARNING
>
> Understanding the next paragraph is crucial to understanding everything that follows.

I saw as in a vision that $P_n NB$ was a ready-made right spiritual triangle with two known sides: NP_n is the observer's latitude and BP_n is is the sun's co-declination. Knowing these two variables I could solve for the angle NP_nB, which I call r. And $180° - r = t$, the meridian angle. After solving t, I could then solve for the azimuth, ~~the~~ side $N\underline{B}$ (or the angle Z). That has been my approach, and my auxiliary triangle $P_n NB$ shall henceforth be called "Criswell's Triangle."

Again, Criswell's Triangle is constructed by dropping a line ~~perpendicular~~ from the celestial pole perpendicular to the horizon. The right angle is formed there, by the bottom leg of the figure, or that segment of horizon from the north point, N, to the sun, B. The hypotenuse is the line from the sun to the north celest. pole.

(Note that the hypotenuse is Co-d only for north declinations. For south declinations it is 90° + d.)

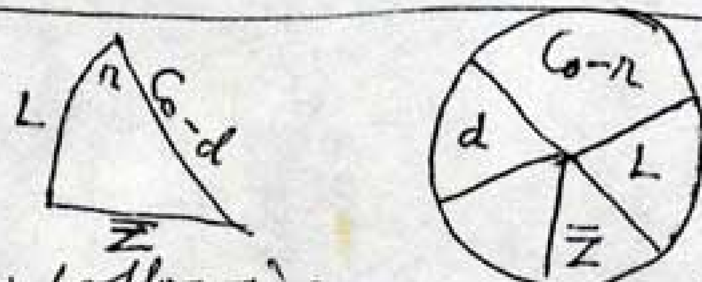
For the sake of completeness I will restate Napier's Rule and draw a Pie.



The sine of a part is the product of (1) the tangents of ~~the~~ its adjacent parts or (2) the cosines of its opposite parts.

In my derivation I will substitute Astro Triangle labels for A, B, C, a, b and c, but the above configuration will remain the "template of reference."

Criswell's Triangle:



(To distinguish the azimuth side of the figure from the angle Z and the apex Z, I label it $\overline{Z}$.

From this I derive the following equations:

$$\cos r = \tan d \tan L \qquad [a]$$
$$\sin \overline{Z} = \cos d \sin r \qquad [b]$$

This pair of equations actually gives all the info needed, but it lacks elegance. The extraneous "r" clutters it up, and it requires an extra step to get t:

$$180 - r = t$$

To correct this I reason as follows:

$$\cos r = \tan d \tan L$$
$$t = 180 - r$$

But $\cos r = -\cos(180 - r) = -\cos t$

The sin-cos graph shows that this is so:

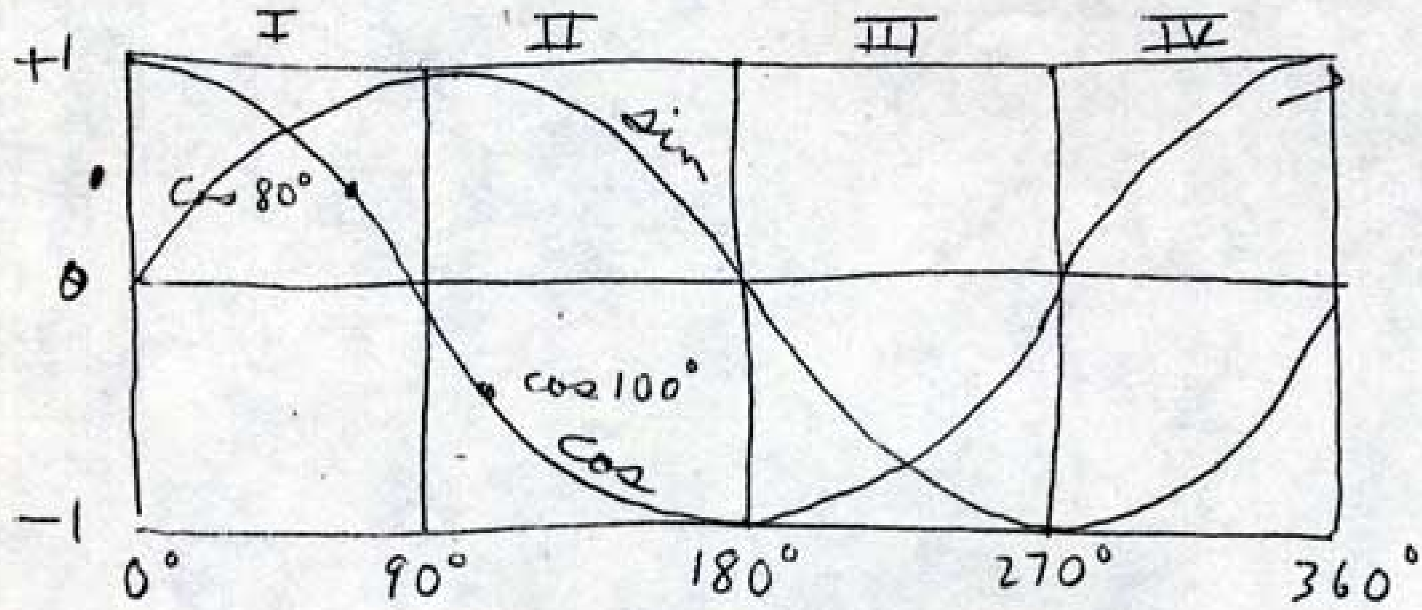


Figure 3A

Say $r = 100°$. $\cos 100° = -\cos(180 - 100)$.

Therefore by simply adding a minus sign to one side of equation [a] we can substitute t for r:

$$\cos t = -(\tan d \tan L) \qquad [I]$$

A formula in Bowditch for meridian angle of a body on the celestial horizon (altitude 0°) is

$$\cos t = \tan d \tan L$$

But if this equation is used as-is it produces an angle $t < 90$ for north declinations and $t > 90$ for south declinations.

Figure 4 shows that this is just the opposite of what is required.

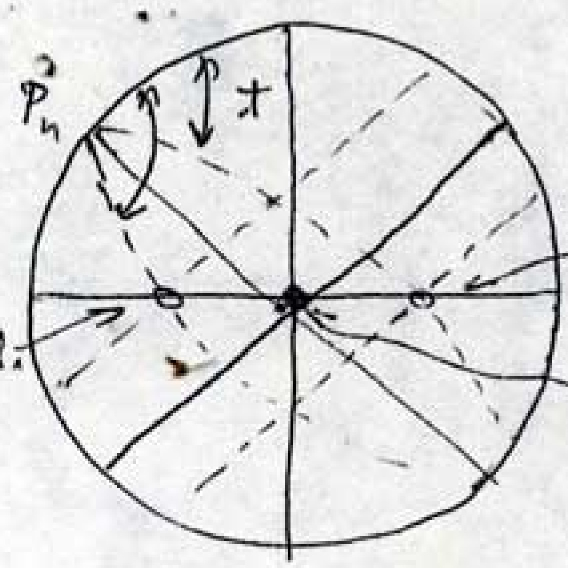


Figure 4.

However, Bowditch's text on Napier's Rules states the following:

"The following rules apply: . . .
2. Side c (the hypotenuse) is less than 90° when a and b are in the same quadrant [⊙ in N. decl., see fig 4], and more than 90° when a and b are in different quadrants. [⊙ in S. decl., see fig. 4] . . .

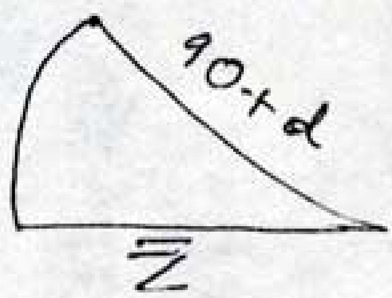
If the rule requires an angle of more than 90° and the solution produces an angle of less than 90°, subtract the solved angle from 180°."

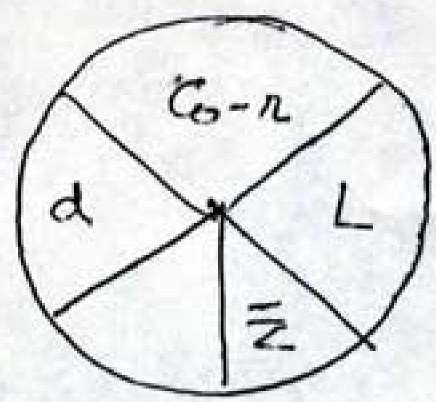
(9) (Revised)

If we follow that rule, $\cos t = \tan d \tan L$ will work, and therefore this is actually the equation I derived. But the minus sign gives the correct answer in one step, so I leave it as ~~is~~ is, to wit:

$$\cos t = -(\tan d \tan L) \qquad [I]$$

Now for south declination, fig 4 shows that the hypotenuse of Criswell's Triangle becomes $90° + d$ (which has a cosine less than 0, see sin-cos graph). But the Pie remains the same:





because $Co(90 + d) = d$

Isn't it beautiful? Don't you think the beauty and ~~symmetry~~ symmetry of the mathematics echos that of the natural phenomena they describe?

Now for azimuth, you remember, I had

$$\sin \bar{Z} = \cos d \sin r. \quad [b] \text{ (page 6)}$$

But $r = 180 - t$ and the sin/cos graph shows that $\sin r = \sin t$. Thus I arrive at my final azimuth formula:

$$\sin \bar{Z} = \cos d . \sin t \qquad [2]$$

– and this, to my surprise and delight, matches a formula in Bowditch for azimuth of a body at any altitude, not just on the horizon.

So I end up with

$$\cos t = -(\tan d \tan L) \qquad [1]$$
$$\sin \bar{Z} = \cos d \sin t \qquad [2]$$

And that's the end of my derivations. I hope you're still with me.

A couple of run-throughs will show the amplitude ($\bar{A} = 90 - \bar{Z}$) always hits Bowditch's Table 27 values (as well as our rise-set program which uses Bowditch's formula) dead on the nose. Since eq. [2] uses t from ~~eq~~ eq. [1], it seems to me that eq. [1] must be correct.

Chapter III
In Which the Sun Finally Rises

So now we have the meridian angle t which the sun at declination d traverses from the observer's celestial horizon to his celestial meridian in latitude L. (Or for sunset, from his C.M. to his C.H.) In this chapter we'll ignore corrections to the visible horizon.

Converting to time units, we have $t/15$ hours from dawn (as defined above) to Local Apparent Noon.

A word about the sun. ~~Time~~ Clock time is measured by the "mean sun", an imaginary body which moves along the Celestial Equator at a constant rate, namely 15° per hour. The actual, or "apparent" sun moves along the ecliptic at an uneven rate since the earth's rotation is subject to various gravitational perturbations; so that the apparent sun is usually ahead of or behind the mean sun. The difference is called the "Equation of Time" (Eq.T.).

But we don't have to worry about this because it's tabulated in the World Almanac under "Sun on Meridian" (Eq. T. = 12:00 − Tabulated time.) This refers to the Greenwich Meridian, but the increments are so small from one day to the next that I consider interpolation to the observer's time zone to be unnecessary. All you have to do is correct for the difference between your longitude and that of your time zone meridian, in our case 90° W.

The time correction for longitude is

$$\frac{TZM - \lambda}{15}$$

where TZM = time zone meridian
λ = Observer's longitude

Your longitude is $83.71^\circ W$, so

$$\frac{90 - 83.71}{15} = \frac{6.21}{15} = 0.41\ h = 24^m 36^s$$

We can redraw Fig 2 now as a Time Diagram:

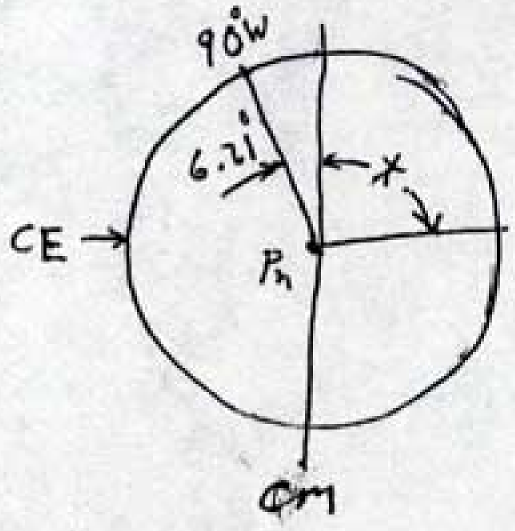


Figure 5. Time Diagram

This shows that the apparent sun is on your meridian $24^m 36^s$ before it gets to the TZM.

Therefore the CST of sunrise = "Sun on merid." value $- 24^m 36^s - \frac{x}{15}$
CST of sunset = "Sun on merid." value $- 24^m 36 + \frac{x}{15}$

Again, this for the center of the sun on the celestial horizon

Chapter IV
In which a Shortcut to the Visible Horizon is Cleverly Devised

To get the sun down onto the visible horizon we need to correct for

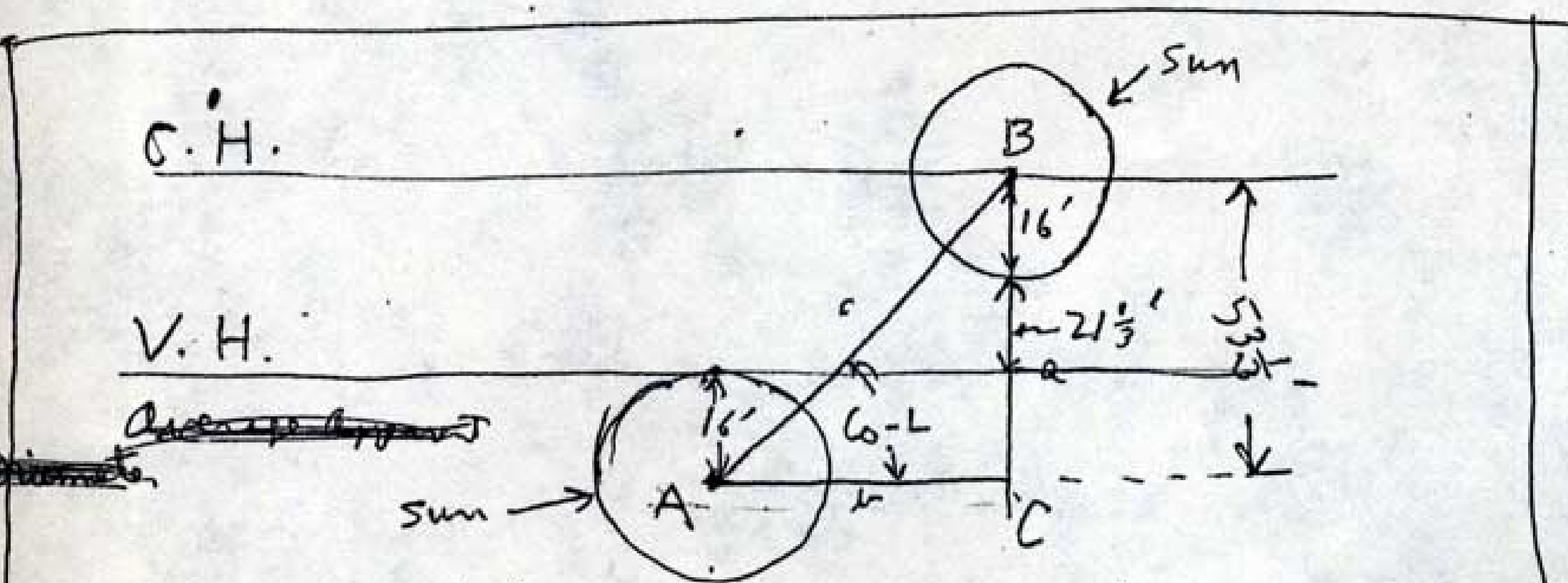
dip	—
refraction	$-34.'5$
irradiation	$+0.'6$
parallax	$+0.'1$
semidiameter	$+16.'0$

But all that is a lot mind-tangling work, and I don't want to lose your attention now as we approach the climax of this investigation. So I'll put it in an appendix, along with my derivations of the two equations needed to correct t and Z, and you can take it or leave it.

In the meantime, I've devised a short-cut based on a statement in Bowditch, which shows the combined effect of all the corrections:

"When the center of the sun is on the celestial horizon, its lower limb is about 2/3 of its diameter above the visible horizon".

Of course this presupposes a flat, unobstructed horizon, like the sea or a plain, and it would vary with height of eye (dip). But unless you're observing from an airplane or a skyscraper, the dip correction only amounts to a few seconds of time, I think. (But we'll examine that in Appendix A.)

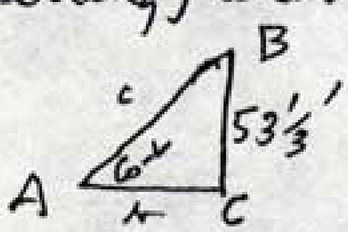


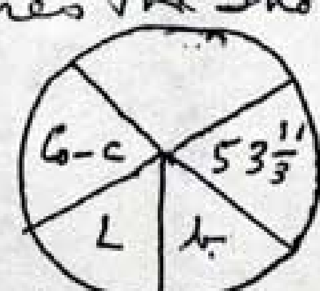
Average apparent diameter of sun = D = 32'

$\frac{2}{3}D = 21\frac{1}{3}'$

Figure 6. The Shortcut to VH.

I know that angle A is not exactly Co-L, ~~but it's close enough for this shortcut~~ unless the sun is at equinox but it's close enough for this shortcut. In Appendix A you can see how I've computed the actual angle of the rising ~~sun~~ (or setting) sun. Here's the Shortcut pie:





$\sin 53\frac{1}{3}' = \sin c \cos L$

$\sin c = \frac{\sin 53\frac{1}{3}}{\cos L} = \frac{\sin 0.89}{\cos L}$

So the upper limb should touch the ~~horizon~~ visible horizon $c/15$ hours ($\frac{60c}{15}$ minutes) before its center reaches the C.H.

Side b is the azimuth correction: $\sin b = \tan L \tan 53\frac{1}{3}'$!

Now let's see if it works.

Chapter V

In Which An Actual Sunrise is Predicted Before the Reader's Very Eyes

Sunrise at Toad Hall, Friday, June 25, 1987

Lat. $34.^{\circ}46$ N

Long. $92.^{\circ}72$ W

☉'s declination $+23^{\circ}25' = 23.^{\circ}42$ (from Almanac)

☉ on TZ meridian (90°W) at 12:02:30 (from Almanac)

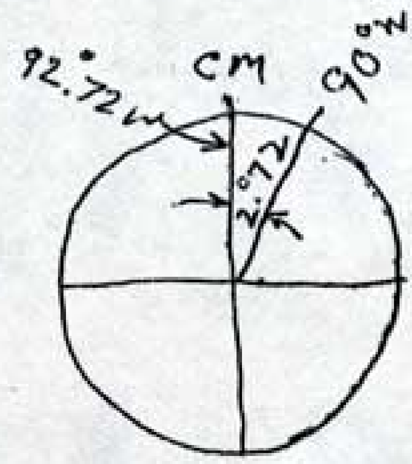
$\cos t = -(\tan 23.^{\circ}42 \tan 34.^{\circ}46)$

$t = 107.^{\circ}29$

$\sin Z = \cos 23.^{\circ}42 \sin 107.^{\circ}29$

$Z = 61.^{\circ}18$

$A = 180 - Z = E28.^{\circ}82N$ (This checks Bowditch Table 27 on the nose)



The sun is on my meridian

$\frac{2.^{\circ}72}{15} = 0.18\text{h} = 10^{m}53^{s}$ after

it crosses the TZ meridian,

or at

$$\begin{array}{r} 12:02:30 \\ +\quad 10:53 \\ \hline 12:13:23 \end{array}$$

So $t/15$ hours earlier it's on the C.H.

$\frac{t}{15} = 7^{h}09^{m}10^{s}$

$$\begin{array}{r} 12:13:23 \\ -\ 7:09:10 \\ \hline 5:04:13 \end{array}$$ CST or 6:04:13 CDT

Correct that for V.H.: $\sin c = \sin 0.89 / \cos 34.46$

$c = 1.^{\circ}08$

$c/15 = 0.07^{h} = 4^{m}19^{s}$

Time of sunrise: 6:04:13 − 00:04:19 = <u>5:59:54 CDT</u> !

The newspaper says 5:57. So I'm $2^m 54^s$ off of that. Who's right? I couldn't tell by observation because of that goddamned hill behind my house.

The correction to the amplitude ($\bar{A} = 28.°82$) is

$$\sin b = \tan 34.46 \tan 0.89 \qquad \text{(page 15)}$$
$$b = 0.°61$$

And the corrected amplitude is

$$28.82 + 0.61 = 29.°43$$

or $E\ 29°26'\ N$

or on the compass, $N\ 60°34'\ E$

Anyway, that's my system. Even though it has taken me 16 pages to explain it (with still more to come), I think you'll agree it's relatively quick and simple to execute. I hope you like it.